ABSTRACT
It is very important to estimate the signal to noise ratio (SNR) of received signal and to transmit the signal effectively for the modern communication system. The performance of existing non-data-aided (NDA) SNR estimation methods are substantially degraded for high level modulation scheme such as M-ary amplitude and Amplitude phase shift keying (APSK) or quadrature amplitude modulation (QAM). In this paper, we propose a SNR estimation method which uses zero point auto-correlation of received signal per block and auto/cross-correlation of decision feedback signal in orthogonal frequency division multiplexing (OFDM) system. Proposed method can be studied into two types; Type 1 can estimate SNR by zero point auto-correlation of decision feedback signal based on the second moment property. Type 2 uses both zero point auto-correlation and cross-correlation based on the fourth moment property. In block-by-block reception of OFDM system, these two SNR estimation methods can be possible for the practical implementation due to correlation based the estimation method and they show more stable estimation performance than the previous SNR estimation methods.

Index Terms:- Auto correlation. OFDM, QAM, SNR estimation.

I. INTRODUCTION
Existing SNR estimators can be classified according to a number of criteria. Data-aided (DA) estimators can be used when the receiver has knowledge of the transmitted symbols, in contrast to non-data-aided (NDA) estimators, which do not require such knowledge. Decision directed (DD) can be used by substituting the true transmitted symbols by the outputs of the decoder. Maximum likelihood estimator is one of DA estimator [1], and squared signal-to-noise variance (SNV), second and fourth order moment (M2M4)-based and signal-to variance ratio (SVR) has been proposed [2-4]. Although ML estimators provide good statistical performance, they tend to be computationally intensive. Under a different classification, I/Q-based estimators make use of both the in-phase and quadrature components of the received signal, and thus require coherent detection; in contrast, envelope based (EVB) estimators only make use of the received signal magnitude, and thus can be applied even if the carrier phase has not been completely acquired. The more signal has high modulation level, therefore, the more SNR estimation is difficult when we compare simple modulation signal such as binary phase shift keying (BPSK) with M-ary amplitude and phase shift keying (APSK) or quadrature amplitude modulation (QAM) modulation signal [5-6]. Even if SNR estimation algorithm could apply efficiently to BPSK signal, there is much difficulty just as it is about high dimensional signal. Therefore, we propose a SNR estimation method based on decision feedback that amenable to practical implementation and significantly improves on previous estimation methods for high level modulation signal.

II. SNR ESTIMATION USING CORRELATION
The received signal at the front end of receiver is
\[ y(n) = x(n) + w(n) \]  
Where \( x(n) \) and \( y(n) \) are transmitted and received signal, respectively. \( w(n) \) is additive noise which is assumed to be zero mean AWGN and uncorrelated with the signal. In this case, the autocorrelation of the measured data, \( y(n) \), is given as,
\[ r_y(k,l) = r_x(k,l) + r_w(k,l) \]  
Assuming \( y(n) \), a wide-sense stationary random process, the autocorrelation \( r_y(k,l) \) depends only on the difference, \( m = k - l \). Thus, (2) may be rewritten as
\[ r_y(m) = r_x(m) + r_w(m) \]  
Because a zero-mean AGWN \( w(n) \) models the nondeterministic part of (1), this process is uncorrelated.
with itself for all lags, except at \( m = 0 \), and its autocorrelation sequence (ACS) has the following form.

\[
\gamma_r(m) = \sigma^2 \delta(m) \quad \text{(4)}
\]

Where \( \sigma^2 \) is the variance of noise, and \( \delta(m) \) is the discrete delta sequence. Since the autocorrelation sequence of \( y(n) \) is a conjugate symmetric function of \( m, \gamma_r(m) = \gamma_r(-m) \) with the amplitude upper bounded by its value at \( m = 0 \). SNR of received signal is defined as

\[
\rho = \frac{\mathbb{E}(|x(n)|^2)}{\sigma^2} \quad \text{(5)}
\]

When we consider transmit signal of random variable in set \{+a, -a\} with equal probability and AWGN channel, auto-correlation value of transmit and received signal are as follows. We can be expressed as signal power \( S \) and noise power \( N \).

\[
\gamma_x(0) = \mathbb{E}[x(n)x^*(n)] = 2a^2 = S \quad \text{(6)}
\]

\[
\gamma_y(0) = \mathbb{E}[y(n)y^*(n)] = 2a^2 + 2\sigma^2 = S + N \quad \text{(7)}
\]

Where auto-correlation value of \( x(n) \) and \( y(n) \) are transmit and receive signal power, respectively. By equation (3) and (4), noise power is \( \gamma_r(0) = \gamma_x(0) - \gamma_y(0) \).

Therefore, SNR based on auto-correlation of equation (5) is given as

\[
\hat{\rho} = \frac{S}{N} = \frac{\gamma_x(0)}{\gamma_x(0) - \gamma_y(0)} \quad \text{(8)}
\]

Type 2 SNR estimation method estimates SNR using zero point correlation relation of received signal based on fourth moment equation (9) and (10) is fourth moment with square of zero point auto-/cross-correlation of transmit and receive signal. Zero point auto-cross correlation are given as

\[
\gamma^2_x(0) = \mathbb{E}[x(n)x^*(n)]^2 = S^2 \quad \text{(9)}
\]

\[
\gamma^2_y(0) = \mathbb{E}[y(n)y^*(n)]^2 = (S + N)^2 \quad \text{(10)}
\]

\[
\gamma^2_{xy}(0) = \mathbb{E}[x(n)y^*(n)]^2 = S(S + N) \quad \text{(11)}
\]

SNR based on fourth moment can calculate as auto-cross correlation relation of transmit and receive signal, and is derived as follows.

\[
\frac{\gamma^2_x(0)}{\gamma^2_x(0) - 2\gamma_{xy}(0) + \gamma^2_y(0)} = \frac{S^2}{S^2 + N^2 - 2S^2 + 2SN} \quad \text{(12)}
\]

Therefore, final SNR of Type II SNR estimation method based on correlation relation can be expressed by

\[
\hat{\rho} = \frac{S}{N} = \frac{\gamma^2_x(0)}{\sqrt{\gamma^2_x(0) - 2\gamma_{xy}(0) + \gamma^2_y(0)}} \quad \text{(13)}
\]

### III. Correlation and Estimation

Data input into mapping block and modulate to complex data symbol such as QPSK or QAM. And, then, serial data stream converts to parallel data and this parallel data pass IFFT (inverse fast Fourier transform). Then, transmit signal of general OFDM (orthogonal frequency division multiplexing)

\[
x(t) = \sum_{k=0}^{K-1} x_k e^{2\pi f_x t} = \sum_{k=0}^{K-1} X_k e^{j2\pi ft} \quad \text{(14)}
\]

Where \( K \) is total sub-carrier number, \( T_s \) is symbol duration, frequency of sub-carrier is \( f_k = \frac{k}{K} T_s \), and \( t \) is \( n \cdot T_s (n = 0, ..., K - 1) \). Also, \( X_k \) is data symbol at \( k \)-th sub-carrier. Transmit signal \( x(t) \) can be expressed to discrete signal as follows.

\[
x(n) = \sum_{k=0}^{K-1} X_k e^{j2\pi f_k n} \quad \text{(15)}
\]

To simplify analysis of system, communication channel assume to AWGN.

\[
x(t) = x(n) \otimes h(n) + w(n) \quad \text{(16)}
\]

Considering AWGN channel to analyze mathematically, channel response \( h(n) \) equals to 1 and phase synchronization supposes to be perfect. After removing cyclic prefix, after FFT, the recovered output for the \( k \)-th sub-carrier is as follows:

\[
Y_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N} = X_k + N_k \quad \text{(17)}
\]

**Figure 1 SNR estimation method in OFDM system.**

SNR estimation method of this paper requires zero point auto-correlation of transmitted and received signal in OFDM system. In this paper, we can calculate autocorrelation value of transmit signal using decision feedback signal. In OFDM system, auto-correlation of transmit signal with QPSK and QAM constellation is calculated as follows.

\[
\gamma_x(0) = \gamma_x(0) = \frac{1}{K} \sum_{k=0}^{K-1} |X_k|^2 = 2 \quad \text{for QPSK} \quad \text{(18)}
\]

For 16QAM signal

\[
\gamma_x(0) = \frac{1}{K} \sum_{k=1}^{K} |X_k|^2 + \sum_{k=1}^{K} |X_k|^2 + \sum_{k=1}^{K} |X_k|^2 \quad \text{(19)}
\]

For M-QAM signal

\[
\gamma_x(0) = \frac{1}{K} \sum_{k=1}^{K} |X_k|^2 + \sum_{k=1}^{K} |X_k|^2 + \sum_{k=1}^{K} |X_k|^2 \quad \text{(20)}
\]

Where \( N_{c1}, N_{c2} \) and \( N_{c3} \) are number of symbol of each level in QAM signal with multilevel constellation. And \( X_{nc} \) is data symbol of each level. Auto-correlation value of QPSK signal is always 2 regardless of random variable signal, (i.e., although error of decision signal is included in receiver side). Therefore, auto-correlation of transmitted M-QAM signal at receiver side is impossible to predict.
Figure 2 Efficient SNR Estimation In OFDM Systems.
We use auto-correlation value of decision feedback signal instead of transmit signal. In case SNR estimates through separated in-phase and quadrature signal, auto-correlation value of decision feedback signal with M-QAM constellation at receiver side is as follows.

For 16QAM signal,
\[
\Delta r_x(0) = \left( \sum_{s=1}^{2^{2i+1}} (X'_t)^2 - \sum_{s=1}^{2^{2i+1}} (X'_q)^2 \right) + \left( \sum_{s=1}^{2^{2i+1}} (X'_q)^2 - \sum_{s=1}^{2^{2i+1}} (X'_t)^2 \right)
\]

Where \( r_x(0) \) is auto-correlation value of decision feedback signal equation (22) shows case of 16-QAM. In equation (22), auto-correlation value \( r_x(0) \) of 16QAM has difference in case that \( \pm 1 \) is decided to \( \pm 3 \) and in case that \( \pm 3 \) is decided to \( \pm 1 \). Here, \( N \) is number of sub-carrier, \( E_{np}^{2i+1} \) is number of symbol which 1 is decided to 3, and \( X'_r \) is a symbol of that time. In contrast, \( E_{np}^{2i+1} \) is number of symbol which 3 is decided to 1. Because both \( \pm 1 \) and \( \pm 3 \) is calculated as absolute value, error symbols which 1 becomes 3 are increased auto-correlation value and errorsymbols which 3 becomes 1 are decreased auto-correlation value. Therefore, difference of auto-correlation between transmit signal and decision feedback signal can be generalized as equation (23) and (24).

For 16QAM signal
\[
\Delta r_x(0) = r_y(0) - r_x(0)
\]

\[
= \left( \sum_{s=1}^{2^{2i+1}} (X'_t)^2 - \sum_{s=1}^{2^{2i+1}} (X'_q)^2 \right) + \left( \sum_{s=1}^{2^{2i+1}} (X'_q)^2 - \sum_{s=1}^{2^{2i+1}} (X'_t)^2 \right)
\]

Figure 3 Generating random input data of size 1 x 128
Type 2 method estimates SNR using cross-correlation of decision feedback signal. So, in this case, error symbol has an effect to cross-correlation value and SNR estimation performance can be changed. If SNR estimation method based on correlation relation is used, system can benefit synchronization secure or offset estimation as well as SNR estimation. As we show difference of auto-correlation between transmit and decision feedback signal, in previous section, cross-correlation between received and decision feedback signal can be organized as follows.

\[
\hat{\rho} = \frac{[\gamma(0) + \Delta\gamma(0)]^2}{\gamma(0)^2 - 2[\gamma(0) + \Delta\gamma(0)]^2 + [\gamma(0) + \Delta\gamma(0)]^2}
\]

for type 2 - (27)

IV. SIMULATION RESULTS

Firstly, we use the MSE (mean squared error) to evaluate the performance of SNR estimation algorithm. The best SNR estimator is unbiased (or exhibits the smallest bias) and has the smallest variance. The statistical MSE reflects both the bias and the variance of an SNR estimate and is given by

\[
MSE(\hat{\rho}) = E((\hat{\rho} - \rho)^2)
\]

where \(\hat{\rho}\) is an estimate of the SNR and \(\rho\) is the true SNR. And we compare estimated performance and MSE with existing considerable NDA estimators; moment-based SNR estimation method of second and fourth moment (M2M4) [3-6], and six moment (M6) [7]. These methods belong to the class of NDA envelope based (EVB) estimators, requiring neither accurate carrier recovery, nor knowledge of the transmitted symbols. This flexibility, together with implementation simplicity, makes these estimators attractive for practical applications.

Figure 2 shows mean SNR estimate performance of ideal and experimental value for proposed SNR estimation method in 16QAM-OFDM system. In 4QAM or QPSK, proposed method is the same performance between ideal and experimental results because correlation relation of decision feedback signal doesn’t change from transmit signal’s one. But the higher modulation level, estimation error is bigger. In Figure 2, i.e. in case of 16QAM, proposed method shows about 0.5dB difference from ideal case because of correlation value with errors of decision feedback signal.

V. CONCLUSION

We proposed a correlation relation-based approach that is amenable to practical implementation and significantly improves on previous estimators. Proposed method of this paper showed stable performance than previous SNR estimation method because this estimation method uses zero point auto-correlation of received signal per block and auto-cross correlation of decision feedback signal in OFDM system. Proposed SNR estimation method had similar performance with CRLB for QPSK and QAM and had NMSE under 0.005 in wide SNR range of from -10dB to 30dB. Especially, Type 1 method had an estimation error under 2dB even though the signal for less than 0dB and NMSE performance of CRLB. Type 2 method has a NSER performance under 0.005 for more than 10dB SNR. Due to its simplicity and practicality, therefore, proposed method is an attractive choice, which recently proved competitive for high level modulation.
REFERENCES


